

The Liabilities and Risks of State-Sponsored Pension Plans

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Most U.S. state governments offer their employees defined benefit pension plans. This arrangement contrasts with the defined contribution plans that now prevail outside the public sector, such as 401(k) or 403(b) plans, in which employees save for their own retirement and manage their own investments. In a defined benefit pension plan, the employer promises the employee an annual payment that begins when the employee retires, where the annual payment depends on the employee's age, tenure, and late-career salary.

When a state government promises a future payment to a worker, it creates a financial liability for its taxpayers. When the worker retires, the state must make the benefit payments. To prepare for this, states typically contribute to and manage their own pension funds, pools of money dedicated to providing retirement benefits to state employees. If these pools do not have sufficient funds when the worker retires, then the states will have to raise taxes or cut spending at that time, or default on their obligations to retired employees.

As of December 2008, state governments had approximately \$1.94 trillion set aside in pension funds. How does the value of these assets compare to the present value of states' pension liabilities? Just as future Social Security and Medicare liabilities do not appear in the headline numbers of the U.S. federal debt, the financial liability from underfunded public pensions does not appear in the headline numbers of state debt. Government pension accounting should ideally provide citizens and government officials with a sense of how indebted the taxpayers are to

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state employees. If pensions are underfunded, then the gap between pension assets and liabilities is off-balance-sheet government debt.

We show that government accounting standards require states to use procedures that severely understate their liabilities. States project the payments they owe to retirees, but in calculating how much those payments are worth today, the states use discount rates that are unreasonably high. In particular, government accounting standards require them to discount their liabilities at the expected return on their assets. This approach is analytically misguided: the magnitude of pension liabilities and how a pension's funds are invested are two separate issues that should be considered independently. In practice, the accounting standard being used sets up a false equivalence between pension payments, which are extremely likely to be made, and the much less certain outcome of a risky investment portfolio.

We begin this article by discussing the true economic funding of state public pension plans. Using market-based discount rates that reflect the risk profile of the pension liabilities, we calculate that the present value of the already-promised pension liabilities of the 50 U.S. states amount to \$5.17 trillion, assuming that states cannot default on pension benefits that workers have already earned. Net of the \$1.94 trillion in assets, these pensions are underfunded by \$3.23 trillion. This "pension debt" dwarfs the states' publicly traded debt of \$0.94 trillion. We show that even before the market collapse of 2008, the system was economically severely underfunded, even though public actuarial reports presented the plans' funding status in a more favorable light. While we take no stand regarding the optimal amount of state government debt, we do believe it is important to point out that total state debt with pension liabilities included is actually almost 4.5 times the value of outstanding state bonds.

A related question is whether taxpayers should be concerned about the fact that state pension funds are invested in risky assets. Under current pension fund investment policy, there is a wide distribution of possible future funding outcomes. The outcomes are skewed in such a way that there is a small probability of an extremely good outcome and a large probability of poor outcomes. We review some theoretically plausible reasons why current taxpayers might not care about this distribution. We are skeptical, however, that the necessary conditions for the irrelevance of state pension fund investment policy hold. We provide the distribution of outcomes so that taxpayers can decide for themselves whether the state is taking an acceptable level of risk on their behalf.

It is important to emphasize that state defined benefit pension plans and individual defined contribution pension plans have different objectives. An individual 401(k) or 403(b) plan is a savings vehicle for an individual. Optimal asset allocation in such plans is governed by the maximization of individual lifetime utility (Campbell and Viceira, 2002). A state defined benefit pension plan serves the purpose of delivering a contractually pre-specified annuity for the state employees, with taxpayers on the hook for shortfalls.

What is the True Funding Status of Public Pension Plans?

In this section, we analyze the true funding status of state public pension plans, with the underlying technical material drawn from our paper Novy-Marx and Rauh (2009). There are two main subtleties in assessing the funding position of a pension system. The first is how to deal with the fact that promised defined benefits for the typical worker increase very slowly when the employee is new, and very quickly when the worker nears retirement. The second is how to discount projected future streams of payments from pension liabilities.

Recognizing Pension Liabilities as Retirement Obligations Accumulate

In a typical defined benefit pension plan, a worker accrues the right to an annual benefit upon retirement that equals a flat percentage of that worker's final (or late-career) salary times the worker's years of service with the employer. As an example, consider a worker in a plan with a 2 percent "benefit factor." The worker has 10 years of job tenure and has an average wage in the last several years of work equal to \$40,000. If this worker quits the job today, the worker would be entitled to a pension of \$8,000 per year ($.02 \text{ per year} \times 10 \text{ years} \times \$40,000$) upon reaching retirement age, plus any cost-of-living adjustments the plan offers. Such cost-of-living adjustments vary by plan, but typically they either assume a fixed rate of inflation (for example, 3 percent) or are tied to the Consumer Price Index (Peng, 2009). Notice that for a given worker, both the years of service and the salary will grow with each year of work, so that the nominal retirement benefit that a worker expects to receive increases more than proportionately with the worker's age.

One approach for a state is to view the obligation to the worker as fully funded if the fund could deliver an annuity that would cover all retirement benefits that have already been earned. In other words, the state could view its pension liability as though all of its workers were going to quit work today, wait until the retirement age, and collect their promised benefits.

This method is called the Accumulated Benefit Obligation (ABO) measure. This approach is also sometimes referred to as a "termination liability," because it is approximately equal to the present value of what would be owed if the entire workforce were laid off. This approach also implies that in any given year, the pension liability grows by the cost of an annuity that would cover the additional retirement benefits that the pension formula now grants to its workers.

For a pension plan to be considered fully funded, its assets should at a minimum be equal to the Accumulated Benefit Obligation. However, even if the state has assets to cover the Accumulated Benefit Obligation, one could still view it as having some unfunded obligations, since the Accumulated Benefit Obligation will rise more than proportionately as each employee ages. Other possible measures of obligations take into account some of the increase in benefits expected with future service. For simplicity, however, and to be conservative, we will focus on Accumulated Benefit Obligation funding in this paper.

We collected data on the 116 major pension plans sponsored by the 50 U.S. states from their Comprehensive Annual Financial Reports. Total liabilities stated in the plans' annual reports were \$2.81 trillion as of 2007, and we call this the "stated liability." We estimate that when all 2008 reports are in, the total stated liability will be \$2.98 trillion as of 2008.¹

In the financial reports from state pension funds, most of the liabilities are stated on a slightly broader basis than the Accumulated Benefit Obligation. Most states report actuarial liabilities under the Entry Age Normal method (or a related method). Under this method, new service liabilities accrue as a fixed percentage of a given worker's salary, that is, states recognize the cost of the retirement benefits accruing to a worker in a given year as a constant fraction of the worker's salary. The Entry Age Normal method therefore recognizes more than an Accumulated Benefit Obligation, since Accumulated Benefit Obligation liabilities accrue as a low percentage of salary early in the worker's career. The rest of state liabilities, approximately 15 percent, are calculated under a Projected Benefit Obligation method, which takes projected future salary increases into account, but not future years of service. Hence, the Projected Benefit Obligation is between an Accumulated Benefit Obligation and an Entry Age Normal calculation.²

States' pension obligations would therefore look somewhat larger than what we calculate if we used the states' own accrual methods. By putting all of the calculations on an Accumulated Benefit Obligation basis, we are deliberating making a modest estimate of the state's pension obligation.

We estimate that if states were required to report Accumulated Benefit Obligation liabilities, but without any changes to current discounting practices, they would report \$2.87 trillion as of the end of 2008. Based on our asset estimate of \$1.94 trillion, states in aggregate were underfunded by this measure by \$0.93 trillion as of the end of 2008. Note that state accounting standards do not require states to maintain fully funded pension plans, but rather to report pension liabilities using a specified methodology.

Choosing an Appropriate Discount Rate

Most state pension funds use an 8 percent discount rate for converting their expected future pension payments into a present value, and there is very little variation in discount rates across states. The use of 8 percent appears to be a rule of thumb, but it does not have a valid economic motivation. Let's begin by considering what the appropriate discount rate should be.

If pension payments were risk-free, then they should be discounted using

¹ Of the latest Comprehensive Annual Reports of the 116 plans, 66 gave liabilities for fiscal year 2008, whereas the remainder gave them from 2007. We observe that liabilities are growing on average at 6 percent, so we grossed up the liabilities of those plans reporting as of 2007 by 6 percent to arrive at 2008 projections.

² To be more precise, the Projected Benefit Obligation arises from the implementation of the Projected Unit Credit method typically employed by state actuaries.

risk-free interest rates to arrive at a liability measure. The question is how much to credit the sponsor for the possibility that the actual benefits may be higher or lower than expected due to a variety of factors, including uncertainty about future wages, inflation, mortality, retirement dates, and rates of job leaving—and also the possibility that the state can renege on the promised payments.

Consider payments that the states have promised employees for years of work already done—that is, the payments that give rise to the basic Accumulated Benefit Obligation liability. From the state's point of view, these cash flows are extremely likely to be incurred. First, state constitutions in many cases provide explicit guarantees that public pension liabilities will be met in full (Brown and Wilcox, 2009). Second, state employees are a powerful constituency, making it hard to imagine that their already-promised benefits would be impaired. Third, the federal government might well bail out any state that threatened not to pay already-promised pensions to state workers. In practice, Accumulated Benefit Obligation pension liabilities are probably the most senior of all unsecured state debt.

In contrast, consider future benefit accruals. Given the difficulties of state pension funds described throughout this article, state workers would be unwise to assume that all future retirement benefits will accumulate according to the existing formulas. After all, states can change the benefit formula for future accruals. From the state's point of view, these pension obligations that have yet to arise can be trimmed.

Standard financial theory suggests that financial streams of payment should be discounted at a rate that reflects their risk, and in particular their covariance with priced risks. In the case of state pension funds, the “risk” is the level of certainty as to whether certain payments will need to be made. From this point of view, the right discount rate for Accumulated Benefit Obligation pension liabilities is not 8 percent, a rate which implicitly assumes a high covariance with the market, but rather a risk-free interest rate, like the interest rate on Treasury bills and bonds.

How much difference does it make if states used a risk-free rate instead of the typical 8 percent to discount future pension liabilities? We have modeled the prospective stream of payments from state pension promises using each state's stated liability, stated discount rate, and actuarial cost method, as well as information on benefit formulas, the number and average wages of state employees by age and service, salary growth assumptions by age, mortality assumptions, cost-of-living adjustments, and separation (job leaving) probabilities by age. We then use interest rates on Treasury securities as of January 2009 to discount the projected cash flows implied by Accumulated Benefit Obligation pension promises. We find that total liabilities were \$5.17 trillion as of the end of 2008, implying that the underfunding in state pension plans net of the \$1.94 trillion in assets is \$3.23 trillion. This underfunding number assumes only the payments included in the Accumulated Benefit Obligation, and would be larger under any broader accounting measure.

The \$3.23 trillion of unfunded pension-related debt can also be expressed on a per-participant and per-taxpayer basis. There are approximately 20 million individuals who have earned benefits under state pension plans as current or former

employees, and are either receiving benefits now or expecting to receive them in the future. On average, the unfunded pension debt owed to each of these plan participants is therefore \$161,500 (or \$3.23 trillion divided by 20 million). Based on an approximate 2008 U.S. population of 304 million individuals, the pension underfunding works out to \$10,625 for every man, woman, and child in the United States. The \$3.23 trillion pension underfunding can also be thought of as amounting to around \$21,500 for each of the approximately 150 million households that filed tax returns with the Internal Revenue Service in 2008.

It could be argued that since the current state pension system is unsustainable, state constitutions may be changed so that it becomes easier to default on currently owed pension benefits—which would also justify using a higher discount rate than the risk-free rate. But this outcome seems unlikely. Brown and Wilcox (2009) give a number of explicit examples from recent history where municipalities defaulted on their debt but fully preserved employee pensions. Assuming that there will be no default on Accumulated Benefit Obligation pension benefits *does* allow for the possibility that future pension benefits may not be earned by the same formula, so calculations using this assumption offer a useful yardstick for the measuring the extent to which pensions are funded.

We also note that U.S. Treasury debt trades at a price premium due to its liquidity, which may give it a lower yield than a simple risk-free rate. Pension obligations are nowhere near as liquid as Treasuries. Therefore a liquidity price premium should ideally be removed from Treasury rates before using them to discount default-free but illiquid obligations. Working in the opposite direction, long-term Treasuries may have higher yields due to an inflation risk premium. Pension obligations are at least partially hedged against inflation risk through cost-of-living adjustments that are tied to consumer price inflation. Given the lack of consensus over the relative size of the liquidity price premium and inflation yield premium, we use unadjusted Treasury rates to calculate our default-free liability measures.

We have also made calculations using state-specific taxable bond rates as the discount rate for already accumulated pension obligations. The main advantage of this approach is that a situation in which states would be forced to default on their pension obligations is presumably much the same situation in which states would default on their bonds. Workers themselves seem to value their pension claims based on the probability that they will receive them.³ A further advantage of the taxable bond rate is that it does not contain the liquidity premium embodied in Treasury debt.

It is useful to consider the thought experiment in which a state decides to fund its pension obligation by paying off the beneficiaries today with a portfolio of bonds that generates the same stream of payments as the promised benefits and defaults in the same states of the world with the same recovery rates. Unlike coupons on

³ This is evidenced for example by a willingness to exchange large, risky claims for smaller, safer claims, as in the case of the Canadian Auto Workers Union's 2009 negotiations with General Motors.

state bond debt, state pension benefits are not tax exempt for beneficiaries. Therefore, the state would need to deliver to the pension beneficiaries *taxable* bonds that generate the same stream of payments as the pension benefits and default in the same states of the world. In practice, we gross up the yield on state bonds (also known as “state municipal bonds” or “state munis”) by 25 percent to obtain taxable yields.⁴ We then discount the projected cash flow stream from state pensions at state-specific taxable municipal bond rates. Using interest rates on state bonds as of January 2009 and excluding the tax preference, we calculate a liability of \$3.25 trillion and an aggregate underfunding of the Accumulated Benefit Obligation liability of \$1.31 trillion.

However, it is important to emphasize that using taxable state municipal bond discount rates, as opposed to the risk-free Treasury rate, essentially credits states for the possibility that they can default on pension liabilities. Thus, it would be highly misleading to use a liability measure arising from this method as a benchmark for pension funding. A state with poor credit quality should not set aside less money to fund its pensions simply because it has a high probability of defaulting on its obligations. Taxpayers who are not government employees might actually be comforted, however, by the possibility that states might default, as it lowers the expected amount by which taxes have to rise to cover the shortfall.

Some States in Dire Straits

Table 1 shows pension underfunding for each of the 50 states relative to the state’s total annual tax revenues and gross state product. Ohio faces the largest burden as a percent of total tax revenues. Its total tax revenues in 2007 were \$24.8 billion and its total Accumulated Benefit Obligation pension underfunding using our preferred measure of the risk-free Treasury rates for discounting is \$216.9 billion (\$332.5 – \$115.6). At its current level of tax collection, Ohio would need to devote 8.75 years of tax revenue to pension funding simply to catch up on already-made promises. Of course, Ohio would need additional revenue to fund new benefits that employees earned over that time period, and would need further tax revenue to run state programs other than its retirement systems.

The next four states on the list are Colorado, Rhode Island, Illinois, and Alabama. These states would need to devote 8.3, 7.7, 7.2, and 6.4 years of tax revenue respectively to close the pension gap on promises already accrued by workers.

The state with the biggest absolute level of underfunding is California, with

⁴ Poterba and Verdugo (2008) document that over the ten years from 1998–2007, the spread of Treasuries over municipal bonds has been in the range of 50 to 139 basis points, representing an implicit tax rate of between 14.9 and 30.0 percent. Over the period from 1991 to December 2008, the average implicit tax rate was 26.3 percent, and over the period from 1997–2008 it was even lower. Of course, tax rates of marginal investors change over time, and our use of 25 percent represents only a rough adjustment. Also note that the risk that Congress might change the law to limit the value of the tax exemption may increase municipal yields (Greimel and Slemrod, 1999). If this risk has risen, then one should use a number lower than the Poterba and Verdugo estimate to correct municipal yields for the tax preference.

Table 1
State Underfunding as Percent of Tax Revenues and Gross State Product

<i>State name (# of plans)</i>	<i>Pension assets (\$billion)</i>	<i>Liabilities</i>		<i>Funding status</i>	
		<i>As stated (\$billion)</i>	<i>Using the Treasury rate (\$billion)</i>	<i>Percent of tax revenue</i>	<i>Percent of gross state product</i>
Ohio (5)	115.6	190.9	332.5	-874%	-47%
Colorado (1)	29.3	55.6	105.4	-827%	-32%
Rhode Island (1)	6.0	12.4	27.1	-765%	-45%
Illinois (4)	65.7	151.1	284.8	-717%	-36%
Alabama (3)	22.3	41.0	78.8	-637%	-34%
Wisconsin (1)	62.2	82.9	153.3	-629%	-39%
South Dakota (1)	6.0	7.1	13.6	-603%	-22%
Missouri (3)	27.0	51.3	88.6	-575%	-27%
Mississippi (3)	15.1	29.3	51.8	-573%	-41%
Oregon (1)	46.1	56.6	90.4	-573%	-28%
New Mexico (2)	16.2	26.7	45.0	-554%	-38%
South Carolina (2)	21.8	39.7	68.4	-537%	-31%
Kentucky (3)	21.6	43.6	74.5	-535%	-34%
Oklahoma (4)	12.0	32.3	54.7	-516%	-31%
New Jersey (4)	60.5	123.4	204.8	-496%	-31%
Arizona (3)	25.0	40.6	85.1	-485%	-24%
Connecticut (3)	20.4	42.8	80.7	-469%	-28%
Texas (4)	125.3	179.0	313.5	-467%	-16%
Georgia (2)	53.7	75.2	137.3	-460%	-21%
New Hampshire (1)	4.4	7.8	14.2	-450%	-17%
Maine (1)	8.3	13.7	24.0	-438%	-33%
Nevada (1)	17.8	24.0	44.0	-417%	-21%
Minnesota (4)	36.2	57.9	109.9	-415%	-29%
California (3)	330.0	484.2	805.7	-415%	-26%
Montana (2)	5.9	8.6	15.4	-412%	-28%
Arkansas (3)	8.1	20.8	38.3	-408%	-32%
Louisiana (2)	17.7	35.7	61.4	-403%	-20%
Maryland (1)	27.8	50.2	88.2	-400%	-22%
Hawaii (1)	8.3	16.6	28.1	-389%	-32%
Pennsylvania (2)	70.9	104.1	190.5	-388%	-23%
Iowa (1)	18.1	24.5	42.3	-373%	-19%
Kansas (1)	10.3	20.1	36.0	-372%	-22%
Wyoming (4)	4.8	7.0	12.3	-370%	-24%
Alaska (2)	11.7	14.5	24.3	-366%	-28%
Idaho (1)	8.1	11.9	21.0	-363%	-25%
Utah (3)	18.6	20.4	38.5	-338%	-19%
Indiana (2)	15.5	36.4	62.4	-335%	-19%
Florida (1)	97.2	124.1	213.7	-326%	-16%
Washington (7)	44.3	58.9	101.1	-321%	-18%
Virginia (1)	41.3	61.6	100.1	-317%	-15%
Michigan (4)	43.4	69.9	118.4	-314%	-20%
Massachusetts (2)	37.8	55.4	96.7	-285%	-17%
Tennessee (1)	25.8	34.7	58.1	-284%	-13%
West Virginia (2)	6.6	12.3	19.1	-270%	-22%
New York (3)	189.8	227.0	356.2	-263%	-15%
North Carolina (2)	59.1	68.7	117.0	-256%	-15%

Table 1—continued

State name (# of plans)	Pension assets (\$billion)	Liabilities		Funding status	
		As stated (\$billion)	Using the Treasury rate (\$billion)	Percent of tax revenue	Percent of gross state product
Nebraska (2)	5.4	7.9	14.1	-214%	-11%
North Dakota (2)	2.9	3.6	6.7	-212%	-14%
Delaware (1)	6.2	6.9	12.0	-201%	-10%
Vermont (3)	2.4	3.8	6.7	-171%	-18%
Total (116)	1936.7	2975.1	5167.1	-431%	-24%

Note: Pension liabilities are collected from the Comprehensive Annual Financial Reports (CAFRs) for the 116 largest state public pension plans and adjusted to reflect an Accumulated Benefit Obligation liability discounted using the Treasury yield curve. Pension assets are taken from *Pensions and Investments* for September 2008 and projected forward to December 2008 using asset allocation data and realized asset class investment returns. Tax revenues are from the U.S. Census Bureau *Census of Governments*, and Gross State Product (GSP) is from the Bureau of Economic Analysis. Pension assets and liabilities are aggregated to the state level.

underfunding of approximately \$475 billion by our calculation. California does not make the top five as a percent of tax revenue because it collected \$115 billion of tax revenue in 2007. Vermont, the state with the smallest burden relative to its tax revenues, still would have to dedicate over 20 months of tax revenue at this level to make up for its pension shortfall.

If we were to use a broader measure of pension liabilities that included some allowance for how pension liabilities might accumulate in the future, then the funding status of state pensions would appear worse. For example, on an Entry Age Normal basis, we estimate that the underfunding is about \$500 billion larger than on an Accumulated Benefit Obligation basis. However, this effect would be somewhat mitigated by two factors.

First, future benefits depend on future wages, which may be correlated with the stock market, and because of this correlation it might make sense to discount those payments at a higher rate (Sundaresan and Zapatero, 1997; Lucas and Zeldes, 2006; Benzoni, Collin-Dufresne, and Goldstein, 2007).⁵ Taking this wage risk into account, we find that underfunding on an Entry Age Normal basis could be as little as \$200 billion larger than it would be on an Accumulated Benefit Obligation basis.

Second, states may more easily renege on future benefit payments by freezing pension plans (a so-called “soft default”), since future benefits in many instances do not enjoy the same protection as pension benefits already earned by past years of

⁵ We emphasize that this calculation would only be relevant for broader measures such as the Entry Age Normal and Projected Benefit Obligation, where the sponsor tries to forecast the amount it will owe beneficiaries based on expected future wages and years of service. When pension liabilities are calculated by the Accumulated Benefit Obligation standard, their dependence on the evolution of future wages is removed.

service. A broader measure of pension liabilities that makes allowance for pension benefits earned in the future could turn out to be overstated if it does not account for these two factors.

Why Are States Discounting at 8 Percent?

Current government accounting standards encourage states to fund pension liabilities in such a manner that the expected future value of pension assets should equal the expected future value of pension liabilities. For example, consider the \$2.98 trillion in stated state pension liabilities as of 2008, which have been discounted at 8 percent. Under the simplifying and conservative assumption that these liabilities all come due in 15 years (the approximate duration of state pension liabilities), the rules imply that an investment strategy will fully fund the pension plan if on average the plan will have $\$2.98 \text{ trillion} \times 1.08^{15} = \9.45 trillion in assets in 15 years.⁶ Of course, this approach ignores the fact that any asset allocation strategy generates a distribution of potential outcomes. For example, a highly risky asset allocation strategy might allow pension assets to reach \$9.45 trillion on average, even though the pension funds might be underfunded 99 percent of the time and massively overfunded 1 percent of the time.

However, pension fund accounting rules like those from the Government Accounting Standards Board and the Actuarial Standards of Practice focus on an *expected* value of the investment strategy for a pension fund while ignoring the largely *certain* nature of the pension benefits that have already been earned by past years of work.⁷ Government accounting rules for public pension plans improperly link the asset and liability sides of the plans' balance sheets. If pursued to its logical conclusion, this approach leads to some unpalatable and unreasonable results.

For example, assume that the problem of pension funds as of early 2009 is how to make \$1.94 trillion in current assets be worth \$9.45 trillion in 15 years. For \$1.94 trillion in assets today to be worth \$9.45 trillion in 15 years, they need to grow by 387 percent over the entire period, or 11.1 percent per year. The U.S. stock market had an average return of 11.4 percent from 1927–2008. So based on historical arithmetic returns and the accounting rules that govern pension funds, the state pension funding problem can apparently be solved by having states put 100 percent of their pension fund assets in equities. To put it another way, the accounting cure

⁶ For this example, we are assuming the entire liability comes due with a maturity equal to the duration, which underestimates the sensitivity of the liabilities to discount rates. The single payment "bullet" is less sensitive to falling discount rates than any other distribution of liabilities with the same duration.

⁷ In particular, two especially relevant rules here are GASB 25 and ASOP 27. Government Accounting Standards Rule 25, "Financial Reporting for Defined Benefit Pension Plans and Note Disclosures for Defined Contribution Plans," was adopted in November 1994 and is summarized at (<http://www.gasb.org/st/summary/gtsm25.html>). Actuarial Standard of Practice #27, "Selection of Economic Assumptions for Measuring Pension Obligations," was adopted September 2007 and is available at (http://actuarialstandardsboard.org/pdf/asops/asop027_109.pdf).

for the current funding problem for state pension funds, a problem partially created by investing heavily in equities, is to invest still more heavily in equities.

Of course, if only the expected value matters for investing state pension funds, there is a wide range of even riskier investment strategies under which states could call their pensions fully funded while holding substantially less assets than they currently do. For example, under the current accounting standards, state governments could ostensibly meet their obligations using futures contracts on the stock market to maintain a leverage ratio of 10 to 1. The expected annual return of this strategy is roughly 90 percent, so state pension funds would only need to invest about \$750 million today to have a mean asset value of \$9.45 trillion in 15 years time. This strategy “frees up” \$1.94 trillion (essentially all) of assets currently sitting in public pension funds. After paying off all pension obligations along with the entire \$0.94 trillion in state bonds, the states could distribute \$1 trillion, or more than \$3,250 for each of 304 million American men, women, and children—all while maintaining a “fully funded” pension system! This “Modest Proposal” highlights the absurdity of the government accounting rules.⁸ In this highly leveraged investment strategy, the pension system, while funded in expectation, realizes a shortfall at maturity in 15 years with 99.5 percent probability. The shortfall exceeds \$9.2 trillion with greater than 99 percent probability.

A defender of the current system might argue that while a state could in theory abuse this system by raising allocations in risky assets to such a level, in practice many states have used the 8 percent rate of return because it was roughly based in history. If state pension funds generally invested in a 60/40 mix of stocks and bonds, and if stocks have an 11.4 percent expected return (again, their 1927–2008 historical average) and the risk-free rate is 3 percent, then the expected return is indeed roughly 8 percent. However, this argument ignores the insight that the discount rate for liabilities should have nothing to do with how the assets are invested. In our view, states should not be congratulated for the fact that they have not implemented a highly leveraged investment strategy. Rather they should recognize that discounting the liabilities with the 8 percent expected return on their assets is a problem of the same type as the hypothetical example above, albeit on a smaller scale.

Another way to view the current state pension accounting system is that it does not recognize what financial economists call “state pricing,” the fact that the marginal utility of wealth is higher in states of the world where markets perform poorly. This insight helps to explain the irrelevance of the defense that pension fund asset portfolios should return 8 percent on average. The states of the world in

⁸ We use the term “Modest Proposal” in the satirical spirit of Jonathan Swift’s 1729 essay, “A Modest Proposal: For Preventing the Children of Poor People in Ireland from Being a Burden to Their Parents or Country, and for Making Them Beneficial to the Public.” The premise of Swift’s essay was to solve the problem of child poverty by recognizing that children could be eaten: “I have been assured by a very knowing American of my acquaintance in London, that a young healthy child well nursed is at a year old a most delicious, nourishing, and wholesome food, whether stewed, roasted, baked, or boiled; and I make no doubt that it will equally serve in a fricassee or a ragout.”

which the market performs well and the plans are fully funded are exactly those where the representative taxpayer's utility cost of an underfunding is low. The states of the world in which the market performs poorly, and the realized shortfalls are large, are exactly those where the utility cost of an underfunding is high. If the governments invest in assets with high average returns, the probability of underfunding in the future declines, but the underfunding comes when it hurts the most.

We note that current rules contain incentives for states to invest their pension funds in risky assets with higher expected rates of return, as higher expected rates of return allow them to discount liabilities at higher rates. In turn, this arrangement could allow the state to present lower liability estimates to the public. States probably face some limits, set by political economy and the risk of public outrage, on the extent to which they can invest pension funds in risky assets and claim the expected value as a justification. Furthermore, although incentives to invest in risky assets exist under current accounting rules, we cannot conclude simply on the basis of excessively high discount rates and risky investment strategies that state officials are acting on these incentives.

How Did State Pension Funds Reach This State of Affairs?

Part of the current crisis for state pension funds is attributable to the fact that they invest in risky assets that have performed very poorly in the last few years, and especially in the later part of 2008. As of September 30, 2008, state pension funds were invested approximately 53 percent in public equity, 8 percent in private equity, 7 percent in real estate equity, and the remaining 32 percent in fixed income securities, according to *Pensions and Investments* magazine. However, while state pension fund asset values were \$0.4 trillion lower in December 2008 relative to September 2008, and \$0.7 trillion lower relative to September 2006, they were substantially underfunded even before the financial crisis.

Table 2 shows the analysis of pension liabilities and funding status from 2005 through 2008 at the end of each year. Using unadjusted liabilities stated in the Comprehensive Annual Financial Reports, state pensions were underfunded by about \$0.2 trillion in 2005 and 2006, then appeared fully funded in 2007, and then were underfunded by \$1.0 trillion in 2008. Since pension fund assets fell from \$2.8 trillion to \$1.9 trillion (and on a stated basis pensions are now underfunded by \$1 trillion) it at first may appear as though the entire problem in public pension funds is due to the poor market performance of 2008.

Remember, however, that these unadjusted numbers use a discount rate of 8 percent for future liabilities. When we calculate pension fund liabilities using interest rates on Treasury borrowing, we find that liabilities of state pension funds were much larger than stated liabilities even in 2005. Compared to \$2.44 trillion in stated 2005 liabilities, Accumulated Benefit Obligation liabilities discounted using the Treasury interest rates were \$3.76 trillion, while assets in 2005 were \$2.23 trillion. State pensions were therefore already underfunded on this basis by

Table 2

Total Year-End State Pension Liabilities, Assets, and Funding Level (in trillions of dollars)

<i>Liabilities</i>					
Discount Rate	<i>Stated</i>	<i>Stated</i>	<i>Treasury</i>	<i>Taxable Muni</i>	
Method	<i>Stated</i>	<i>ABO</i>	<i>ABO</i>	<i>ABO</i>	<i>Assets</i>
2005	\$2.44	\$2.36	\$3.76	\$3.06	\$2.23
2006	\$2.60	\$2.51	\$3.82	\$3.35	\$2.42
2007	\$2.81	\$2.71	\$4.36	\$3.50	\$2.78
2008	\$2.98	\$2.83	\$5.17	\$3.25	\$1.94

<i>Funding level (liabilities minus assets)</i>					
Discount Rate	<i>Stated</i>	<i>Stated</i>	<i>Treasury</i>	<i>Taxable Muni</i>	
Method	<i>Stated</i>	<i>ABO</i>	<i>ABO</i>	<i>ABO</i>	
2005	−\$0.21	−\$0.13	−\$1.54	−\$0.83	
2006	−\$0.18	−\$0.09	−\$1.39	−\$0.93	
2007	−\$0.03	\$0.07	−\$1.58	−\$0.72	
2008	−\$1.04	−\$0.89	−\$3.23	−\$1.31	

Note: The top panel of the table shows liabilities and assets of the 116 largest state public pension plans. The first column shows liabilities as stated in the state reports, while the next three columns show liabilities calculated according to the Accumulated Benefit Obligation (ABO) method. Liabilities are discounted with state-chosen (“stated”) discount rates, the Treasury yield curve, and state-specific taxable muni (municipal bond) rates using methods from Novy-Marx and Rauh (2009). Assets are shown on the far right. The bottom panel shows the funding level, or pension liabilities net of pension assets.

\$1.54 trillion in 2005. If instead we use taxable municipal bond interest rates for discounting future pension obligations—which reduces the liability to reflect a possibility that states might default even on accumulated pension obligations—underfunding was \$0.83 trillion.

In carrying out similar calculations for state pension fund assets and liabilities over the last several years, an interesting divergence arises. Between 2007 and 2008, asset values of state pension funds dropped by \$0.9 trillion. When one uses the Treasury-interest-rate assumption for 2007 and for 2008, the decline in Treasury interest rates between those two years leads to an additional increase in pension underfunding of \$0.9 trillion between 2007 and 2008 for a total funding deterioration of \$1.8 trillion. In contrast, discounting pension liabilities by the municipal bond interest rates for 2007 and 2008 implies that funding only deteriorated by \$0.6 trillion between those two years. From 2007 to 2008, the market perceived municipal bonds as much riskier; municipal bond rates rose as a result of increased default risk; and so liabilities discounted at muni rates fell. To put it another way,

an increased probability of default on municipal bonds lowers the expected liability from the perspective of taxpayers, because it reflects an underlying assumption that the probability of default on pension liabilities has increased by the same amount.

In sum, the state pension fund situation deteriorated in 2008 as a result of asset markets' dismal performance that year, but state pensions were substantially underfunded on a true economic basis even before those events.

What Might the Future Look Like For State Pension Plan Solvency?

The fact that states sponsor underfunded plans with risky investments generates a distribution of future pension funding outcomes faced by taxpayers. If pension fund assets perform sufficiently well, taxes will not have to rise to meet pension obligations, and taxpayers could even see money returned to them in the form of lower taxes or increased services. If assets do not perform sufficiently well, deficits will have to be remedied with either tax increases or spending cuts.

What does the distribution of future outcomes look like if states continue with current investment policies? We show this distribution in Table 3. To make a plausible (if simplified) estimate of the future distribution, we begin with several assumptions. First, we assume that stocks evolve according to the (standard) log-normal model; that is, we assume that the logarithm of returns each period is normally distributed, with constant mean and variance. We measure asset allocation for state pension funds as of September 2008 and construct a variance-covariance matrix of the asset class returns based on historical data. Together these imply that the volatility of the plans' holdings is about 8 percent per year. By considering the historical excess returns observed in the stock market, we calculate an expected excess return on the pension assets of 3.25 percent to compensate for the 8 percent volatility of the pension funds.⁹ We then look ahead 15 years, from 2008 to 2023, with that time horizon selected because the average time to retirement for current state workers is 15 years.

As shown in Table 3, the median 15-year outcome under current investment strategies is a shortfall of \$2.8 trillion. The 25th percentile outcome is a shortfall of \$3.4 trillion, the 10th percentile is a shortfall of \$3.8 trillion, and the 5th percentile is a shortfall of \$4.0 trillion. There is a less than a 5 percent chance that the current pattern of pension fund investments will meet the needs of retirees in 15 years. Under current state accounting rules, this distribution is deemed to be underfunded by only \$1 trillion.

The second column of the table shows what happens when we project assets forward using the risk-free Treasury interest rate, rather than an expected rate of

⁹ Historically the stock market offers a roughly 6.5 percent risk premium for exposure to equity volatility of 16 percent per year. We assume a Sharpe ratio (the ratio of the expected excess return on the assets to their volatility) of 0.4, which is approximately the Sharpe ratio of the stock market. This implies an expected excess return on the pension assets of 3.25 percent.

Table 3
Distribution of Aggregate State Pension Funding Outcomes in 2023

<i>Percentile of outcomes</i>	<i>Funding level in 2023 (trillions of 2008 dollars)</i>	
	<i>Assets projected forward with 3.25% expected excess return</i>	<i>Assets projected forward with risk-neutral pricing</i>
1%	-4.33	-4.87
5%	-3.99	-4.66
10%	-3.77	-4.52
25%	-3.36	-4.27
50%	-2.80	-3.92
75%	-2.11	-3.49
90%	-1.36	-3.02
95%	-0.83	-2.69
99%	0.26	-1.97

Note: The table shows the distribution of aggregate state pension funding outcomes in 15 years assuming that stocks evolve according to the standard lognormal model. In the first column, we project assets forward using a rate that includes a market-determined expected excess return to compensate for volatility. In the second column, we project assets forward using the risk-free Treasury interest rate.

We estimate a portfolio volatility for the pension funds of approximately 8 percent (see the text). We estimate that the excess return that the plans' assets will pay to compensate for this 8 percent volatility by considering historical excess returns observed in the stock market: a roughly 6.5 risk premium for exposure to equity volatility of 16 percent per year. We assume a Sharpe ratio of 0.4, which is approximately the Sharpe ratio of the stock market. This implies an expected excess return on the pension assets of 3.25 percent.

return that includes a positive market risk premium. With “risk-neutral” pricing, as discussed earlier, the largest shortfalls in pension funding happen when wealth is lowest and, so, the marginal utility of consumption is high. “Risk-neutral” pricing puts higher weights on states of the world in which marginal utility of consumption is higher (Cox and Ross, 1976). Using the risk-neutral pricing assumption, the median outcome is that pension funds face a shortfall of \$3.92 trillion, and even at the 99th percentile, they will face a shortfall of \$1.97 trillion.

Why Do Public Pension Funding and Investment Policy Matter?

The analysis above suggests that state pension funds are both very underfunded and highly exposed to market risks. In this section, we consider the implications of these findings for current and future generations.

Pension Underfunding is Off-Balance-Sheet Debt

Properly accounting for unfunded pension liabilities and treating them as state debt raises total state debt by \$3.23 trillion. The inclusion of unfunded pension liabilities suggests that total state debt is actually almost 4.5 times the \$0.94 trillion in outstanding state bonds. While we believe this is an important fact for taxpayers to know, we make no assertion about the optimal level of pension underfunding. This is ultimately a question about the optimal level of public debt. In many states, constitutional provisions limit the extent of state general obligation debt. For many state governments, therefore, pension underfunding may be an important source of public sector borrowing.

In terms of the intergenerational consequences of state debt, a starting point is the famous doctrine of Ricardo (1820), which postulates the irrelevance for public welfare of financing current spending with debt versus taxes. If households can anticipate the full extent of higher future taxation, they can save more now and, if necessary, bequeath it to their heirs. Under Ricardian equivalence, the amount of spending matters, but how spending is financed does not.

Of course, Ricardian equivalence only holds if the public is aware of the level of total state indebtedness. The current state of government accounting rules obfuscates the true extent of government debt, making it impossible for households to accurately forecast the extent of the necessary intergenerational transfers. In other words, without public knowledge about the extent of pension underfunding, individuals do not know how much to set aside for their children to help them pay off this debt.

Pension Equity Investment Means Taxpayers Are Borrowing from Employees to Invest in Equities

Equity investing inside of public pension funds can be viewed as equivalent to matching liabilities with bonds, and making side bets which entail borrowing money from the states' employees and investing in the stock market. Consider an employee entitled to a one-time, certain \$10,000 benefit in 10 years, and suppose the 10-year Treasury is yielding 3.6 percent. That obligation could be matched by purchasing a 10-year Treasury for about \$7,000 today ($\$10,000/1.036^{10}$), so the present value of the pension obligation must be \$7,000. If the state invests in something else, such as equities, then it's as if the state matched its pension liability by buying a 10-year Treasury for \$7,000, and bought \$7,000 in equities as a speculative investment that it funded by shorting the 10-year Treasury. There is no speculative element in funding the obligation to the employee with Treasuries. Any correct accounting of assets and liabilities will show no change in net obligations if the state moves from holding the \$7,000 in Treasuries to shorting the \$7,000 in Treasuries to fund their equity position. This line of thought illustrates an underlying point: ultimately, whether pensions should be invested in the market is a question of whether the state should be borrowing to invest in equities.

In terms of the intergenerational consequences of pension fund asset allocation, again a starting place is the idea that citizens may be able to undo government

actions. Equity exposure in pension plans passes through to the taxpayers of the state. If the state increases its pension fund exposure to equities, households can rebalance their own portfolios away from equities. This argument is similar to the Modigliani–Miller (1958) theorem of capital structure irrelevance for corporations.

Of course, in order for the public to unwind the government’s position, it must be aware of the full extent of the government’s net equity position. Moreover, true irrelevance requires that investors are capable of fully unwinding the governments’ investments. One segment of the public finance literature postulates that part of the population cannot adjust its private savings in response to government investment. For example, Abel (2001) assumes some households do not hold equities because of fixed costs associated with portfolio diversification, while Diamond and Geanakoplos (2003) assume some households simply lack private savings.

If taxpayers cannot fully undo government investment policy, then it can have real welfare implications. The welfare effects could go either way—that is, either in favor of or against equity investing in the pension fund. As a general point, government policies that aid unlucky generations at the expense of luckier generations can increase overall welfare (Enders and Lapan, 1982; Gordon and Varian, 1988; Bohn, 1999; Gollier, 2008). Ball and Mankiw (2007) show that implementing optimal intergenerational risk-sharing through a social security system requires that the system hold equity (or, alternatively, that benefits be negatively indexed to equity returns). These holdings expose generations that live through periods of slow economic growth to high stock returns that occur during booms that may happen outside their lifetimes.

Assuming one wanted to share risks across generations through equity investment, there is an additional question of whether it is best done through a national social security scheme or in state pension funds. The U.S. Social Security system is legally obligated to invest trust fund assets in special issues of U.S. government bonds, whereas state pension funds hold unrestricted pools of assets. Consequently, to the extent that equity exposure is desirable, it may be more feasible to implement intergenerational risk-sharing schemes through public pension funds than through the U.S. Social Security system as it exists in its current form.

The downside of equity investing is that states face obstacles to effectively implementing such schemes due to the relative mobility of their citizenry, especially those that carry the greatest share of the tax burden. If a state invests heavily in equities and the market performs poorly, then some of its taxpayers, facing larger future tax bills, may leave for states with portfolios that performed better. Similar intuition helps explain the phenomenon of suburban flight (away from urban areas), which was at least in part driven by citizens voting with their feet for lower taxation (Papke, 1987; Ladd and Bradbury, 1988). This risk associated with state equity investment is unspanned by financial markets and cannot be offset by taxpayers using private savings. In sum, if one wanted to share risks across generations through state equity investments, it would have to be done through a national defined contribution scheme that currently does not exist in the United States.

While there may be valid reasons to invest pension assets in equities, there are several fallacies that yield incorrect justifications for doing so. One is the common perception that stocks are less risky in the long run than in the short run, and the belief that since pension funds have long horizons they can ride out the ups and downs of the stock market. This is not generally the case, and for a relevant class of utility functions, taking a sequence of gambles raises overall risk rather than lowers it. The fallacy in the so-called long-run case for equity investing has been studied in Samuelson (1963), Merton and Samuelson (1974), Bodie (1995), and Ross (1999).

A second misperception is the notion that if individuals should have stocks in their 401(k) or 403(b) retirement portfolios, so should state pension funds, on the misguided logic that the state pension fund must face the same objective function as the individual, simply on a collective and large scale. Defined benefit pension funds, however, must provide pre-specified annuities to their employees, whereas individuals are solving a problem of lifetime utility maximization.

Conclusion

The decline in asset markets in 2008 has made the state pension funding problem more apparent, but it is far from being the main cause of the problem. In real economic terms, state pensions were underfunded by \$0.8 trillion even in 2005, when they appeared fully funded according to government accounting standards. Our analysis has several implications for government accounting standards and public policy.

Each state plan currently reports only one actuarial number for its pension liability. This number is of limited usefulness, because it is based on a number of ingredients that are subject to substantial discretion. At a minimum, states should be required to report liabilities under several pre-specified discount rates, such as Treasury interest rates and interest rates on taxable municipal bonds. States should also be required to report the sensitivity of the pension liability estimate to different assumptions. Better still, states could be asked to report projected annual cash flows from accrued and projected pension benefits, which are a key component to calculating their liabilities, thus allowing analysts to apply their own assumptions or to use standardized assumptions across states.

It is worth noting that the same issues also arise for the many municipal and county pension plans in the United States. According to the U.S. Census of Governments, local plans in aggregate held \$0.56 trillion in assets as of June 2007, which is about 20 percent of what state pension plan assets were at the time. According to *Pensions and Investments*, as of September 2008 the largest of these local plans were New York City (\$93 billion in assets), Los Angeles County (\$35 billion in assets), and San Francisco County (\$14 billion in assets). If local plans were as underfunded as state plans, underfunding would be \$0.90 trillion using Treasury discount rates.

Our analysis also highlights for policymakers the perils of focusing only on

average expected outcomes for invested pension fund assets. Distributions of the outcomes of state pension investments will not matter if households can systematically alter their own investment and consumption plans to offset government policy. However, given the lack of transparency of the state pension fund system, households are currently unlikely to understand what such an offset would entail. Furthermore, households face risks that are unspanned by securities markets, such as the possibility that their neighbors will move away and leave them to pay the taxes that will be levied to cover the shortfall. The lack of transparency of state pension fund systems makes it more difficult for taxpayers to optimize their own portfolios and consumption choices over the life cycle.

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